Isospin breaking in πN scattering at threshold by radiative processes

T.E.O. Ericson [a]* and A. N. Ivanov[b] †

[a] Theory Division, Physics Department, CERN, CH-1211 Geneva 23, Switzerland,
[b] Atomic Institute of the Austrian Universities, Vienna University of Technology,
Wiedner Hauptstrasse 8-10, A-1040 Wien and SMI of the Austrian Academy of Sciences,
Boltzmanngasse 3, A-1090, Wien

We investigate the dispersive contribution by radiative processes such as $\pi^- p \to n\gamma$ and $\pi^- p \to \Delta\gamma$ to the πN scattering lengths of charged pions in the heavy baryon limit. They give a large isospin violating contribution in the corresponding isoscalar scattering length, but only a small violation in the isovector one. These terms contribute 6.3(3)% to the 1s level shift of pionic hydrogen and give a chiral constant $F_\pi^2 f_1 = -25.8(8)$ MeV.

PACS: 11.10.Ef, 11.55.Ds, 13.75.Gx, 36.10.Gv

1. Introduction

The energy shift and width in the $\pi^- p$ atom as compared to the purely electromagnetic bound state energies have recently been measured to the remarkable precision of $\pm 0.2\,\%$ and $\pm 1\,\%$, respectively [1] (see also [2]). These quantities are proportional to the corresponding real and imaginary part of the threshold $\pi^- p$ scattering amplitude (the scattering length) [3,4], which therefore are determined to the same precision provided electromagnetic corrections on the several % level can be properly understood and accounted for.

In the absence of the external Coulomb field, the real and imaginary parts of the S–wave amplitude $f^{\pi^- p}(0)$ at threshold ($q = 0$, where q is the relative momentum of the $\pi^- p$ pair) are equal to

$$\mathcal{R}e\, f^{\pi^- p}(0) = a_{cc}\ ,\ \mathcal{I}m\, f^{\pi^- p}(0) = q_{\pi^0 n}\, a^2_{nc}. \quad (1)$$

Here $a_{cc} = a_{\pi^- p \to \pi^- p}$ and $a_{nc} = a_{\pi^- p \to \pi^0 n}$ are the S–wave scattering lengths of the reactions $\pi^- p \to \pi^- p$ (the charged channel (cc)) and of $\pi^- p \to \pi^0 n$ (the charge exchange one), respectively, and $q_{\pi^0 n} = 28.040\,\mathrm{MeV}$ is the relative momentum of the $\pi^0 n$ system[3].

One has then, in principle, an exceptional source for the hadronic scattering lengths which can be directly compared to the large body of phenomenological πN phase shift data as well as to predictions of Chiral Perturbation Theory (ChPT). The electromagnetic corrections to the observed energy level shift are of two types: a) iterated ladders of photon exchange with the intermediate πN state remaining in its ground state and b) corrections corresponding to inelastic intermediate states. The first class of corrections has recently been investigated [6]. These moderate corrections were found to correspond to well understood physical effects to a precision commensurate with the present experimental one. While such effects violate isospin, it is not usual to consider them as genuine isospin breaking. The second group of e. m. corrections are intrinsic to the scattering processes. They produce genuine isospin breaking together with the isospin breaking from the strong interaction itself. In order to reduce the data to pure hadronic interactions one must therefore control these intrinsic e. m. contributions to sufficient accuracy and understand their physics. [4].

*Corresponding author. *E-mail* : torleif.ericson@cern.ch
†*E-mail*: ivanov@kph.tuwien.ac.at . Permanent Address: State Polytechnic University, Department of Nuclear Physics, 195251 St. Petersburg, Russian Federation

[3]We have removed the physical contribution to the imaginary part from the radiative channel $\pi^- p \to \gamma n$, which is accurately known from the Panofsky ratio $P = \sigma(\pi^- p \to \pi^0 n)/\sigma(\pi^- p \to \gamma n) = 1.546(10)$[5].

[4]Another use of the relation (1) is the description of the energy shift in pionic hydrogen in terms of ChPT [7]. In this case terms of different origin are not separated. The e. m. contributions include implicitly the Coulomb terms

Here we investigate this second class of electromagnetic corrections. There are excellent reasons to believe that such electromagnetic processes contribute substantially to the π^-p scattering length. In fact, the dominant contribution (65%) to the 1s width of pionic hydrogen is the radiative capture by the electric dipole transition $\pi^- p \to n\gamma$, the so-called Kroll-Ruderman process [8] (Fig.1). The corresponding radiative width is no less than 8% of the strong interaction shift as observed in the Panofsky ratio [5]. This indicates that the corresponding dispersive term may contribute 4% and even more to the energy shift in pionic hydrogen. This is indeed the case as we will show.

Since the Kroll-Ruderman contribution is generated from the pion p-wave part of the nucleon pole term, one must inevitably consider also other important aspects of the well understood p-wave πN physics. Here the Δ isobar and the $N\Delta$ mass difference are central features with the Δ pole as important as the nucleon one [9]. We therefore include it on an equal footing with the nucleon. Since the main purpose of the present paper is to clarify the physical mechanisms of e. m. isospin breaking in the πN scattering lengths in simple terms, we rely on the heavy baryon limit. This is in accordance with p-wave πN phenomenology for which this approximation describes well the main aspects of the interaction. It will become apparent that the bulk of the isospin breaking occurs already in the limit of a vanishing pion mass. The finite pion mass introduces small, but characteristic, additional terms.

The heavy baryon limit, used in chiral perturbation expansion as well, and the threshold condition lead to great simplifications of the problem. The relevant contribution becomes that of an electric dipole process (E1) due to transition radiation by the absorption or emission of the charged pion. In particular, there is no coupling to the baryon convection current nor to its magnetic moment in this limit, while the threshold condition suppresses radiation produced by changes in the pion convection current.

The leading e. m. isospin breaking effects in low energy πN scattering have previously been discussed in particular using Heavy Baryon ChPT [see Refs. [10] and references therein] as well as using a heavy quark model [11]. Such isospin breaking is implicit in several theoretical studies of the contributions to the 1s energy shift of the $\pi^- p$ atom to leading [12] and next to leading order power counting in an effective field theory(EFT) of QCD+QED [7].

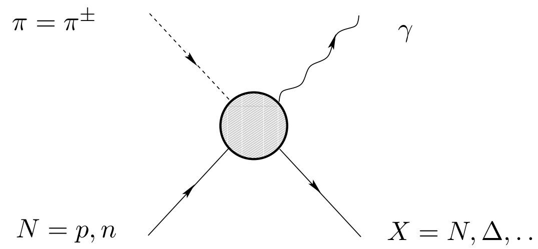

Figure 1. Diagram of $\pi N \to X\gamma$ reactions with one photon in the intermediate state generating a contributiion to low–energy elastic πN scattering.

The paper is organized as follows. In Section 2 we derive the general expression for the dispersive e. m. contributions to the S–wave amplitude of elastic $\pi^- p$ scattering at threshold, saturated by intermediate $X\gamma$ states related to the reactions the $\pi^- p \to X\gamma$, where X is a hadronic state (see Fig. 1). In Section 3 we apply the results specifically to the calculation of the dispersive contributions from the reactions $\pi^- p \to n\gamma$ and $\pi^- p \to \Delta\gamma$, respectively, and generalize this to the elastic $\pi^c N$ threshold amplitude for any charged pion. In Section 4 we discuss the individual contributions to the isospin breaking under different assumptions and discuss the relation of our results to other investigations. In the Conclusion we summarize the results.

2. One–photon exchange contributions to the elastic $\pi^- p$ threshold amplitude

For concreteness we will illustrate these contributions for the case of π^-p elastic scattering at threshold, but the argument is nearly identical for any elastic πN channel with a charged pion. The S–wave amplitude of the $\pi^- p$ scattering caused by the channels $\pi^- p \to X\gamma \to \pi^- p$ with one in-

termediate photon (see Fig. 1) is defined by

$$(1+\frac{m_\pi}{M_N})\,\delta f^{(\gamma)} = \frac{1}{8\pi M_N}\times \lim_{TV\to\infty}\frac{\langle\pi^-(\vec{0}\,)p(\vec{0},\sigma_p)|\mathcal{T}|\pi^-(\vec{0}\,)p(\vec{0},\sigma_p)\rangle}{TV}, \qquad (2)$$

where $TV=(2\pi)^4\delta^{(4)}(0)$ is a 4–dimensional volume and M_N the nucleon mass. The $\mathcal{T}$ matrix is related to the S matrix by $S=1+i\mathcal{T}$, where S is defined by

$$S = \mathrm{T}\exp\Big\{-i\int d^4x\,\sqrt{4\pi}\,e\,J_\mu(x)\mathcal{A}^\mu(x)\Big\}. \qquad (3)$$

Here T is the time–ordering operator, $J_\mu(x)$ the hadronic electromagnetic current [13,14] and $\mathcal{A}^\mu(x)$ the electromagnetic potential.

Integrating over photon degrees of freedom

$$S = \exp\Big\{i\int d^4x\,\mathcal{L}^{\rm em}_{\rm eff}(x)\Big\}. \qquad (4)$$

The effective Lagrangian $\mathcal{L}^{\rm em}_{\rm eff}(x)$ is given by

$$\begin{aligned}\mathcal{L}^{\rm em}_{\rm eff}(x) &= 2\pi\alpha\, i\int d^4y\,\mathrm{T}(J_\mu(x)J_\nu(y))\\ &\times\ \langle 0|\mathrm{T}(\mathcal{A}^\mu(x)\mathcal{A}^\nu(y))|0\rangle \qquad (5)\end{aligned}$$

with

$$\langle 0|\mathrm{T}(\mathcal{A}^\mu(x)\mathcal{A}^\nu(y))|0\rangle = \theta(x^0-y^0)\int\frac{d^3p}{(2\pi)^3 2|\vec{p}\,|} \times e^{-ip\cdot(x-y)}\sum_\lambda e^{*\mu}(\vec{p},\lambda)e^\nu(\vec{p},\lambda) + (x\leftrightarrow y).$$

Here $e^\mu(\vec{p},\lambda)=(0,\vec{e}\,(\vec{p},\lambda))$ is the polarization vector of the photon in the Coulomb gauge. To lowest order in the fine structure constant $\alpha\simeq 1/137.036$, the threshold contribution is

$$\Big(1+\frac{m_\pi}{M_N}\Big)\,\delta f^{(\gamma)} = \frac{1}{8\pi M_N}\times \langle\pi^-(\vec{0})p(\vec{0},\sigma_p)|\mathcal{L}^{\rm em}_{\rm eff}(0)|\pi^-(\vec{0})p(\vec{0},\sigma_p)\rangle. \qquad (6)$$

Substituting Eq. (5) into Eq. (6) and averaging over polarizations of the proton we get

$$\Big(1+\frac{m_\pi}{M_N}\Big)\,\delta f^{(\gamma)} = \frac{\alpha}{8\pi M_N}\int\frac{d^3p}{(2\pi)^2 2|\vec{p}|}\times W_{\mu\nu}(p,k,q)\sum_\lambda e^{*\mu}(\vec{p},\lambda)e^\nu(\vec{p},\lambda)\Big|_{\vec{k}=\vec{q}=0}, \qquad (7)$$

where we have introduced the structure tensor $W_{\mu\nu}(p,k,q)$ as follows:

$$\begin{aligned}W_{\mu\nu}(p,k,q) &= i\int d^4x\,\theta(x^0)\,e^{-ip\cdot x}\\ &\times\frac{1}{2}\sum_{\sigma_p=\pm 1/2}\langle\pi^-(\vec{k}\,)p(-\vec{k},\sigma_p)|\,(J_\mu(x)J_\nu(0)\\ &+\ J_\nu(0)J_\mu(-x))\,|\pi^-(\vec{q}\,)p(-\vec{q},\sigma_p)\rangle. \qquad (8)\end{aligned}$$

For the calculation of $W_{\mu\nu}(p,k,q)$ we insert the complete set of the intermediate hadronic states

$$\sum_X |X\rangle\langle X| = 1,$$

where $|X\rangle$ is an arbitrary hadron state with baryon number $B_X=1$.

We now eliminate the pion field from the current matrix element $\langle X|\,J_\mu|N\pi^\pm\rangle$ following the standard reduction technique in favour of the hadronic axial current $\langle X|J^{(\pm)}_{5\mu}|N\rangle$, where the isospin label $J^{(\pm)}_{5\mu}(x)=J^{(1)}_{5\mu}(x)\pm i\,J^{(2)}_{5\mu}(x)$. This is achieved, for example, using PCAC and minimal electromagnetic coupling together with soft–pion assumptions [13,14]. One finds for the case of $\pi^-p\to X\gamma$:

$$\begin{aligned}&\langle X|J_\mu(0)|\pi^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle =\\ &= -\frac{i}{\sqrt{2}F_\pi}\langle X|J^{(-)}_{5\mu}(0)|p(\vec{0},\sigma_p)\rangle. \qquad (9)\end{aligned}$$

Here $F_\pi = 92.4(3)\,\mathrm{MeV}$ is the pion decay constant. In the soft–pion limit this gives the Kroll–Ruderman theorem [8] (see e. g. [9,14]).

Substitution into Eq. (7) and integration gives the structure tensor in the form

$$\begin{aligned}W_{\mu\nu}(p,k,q) &= \frac{1}{F^2_\pi}\sum_X\Big[\frac{(2\pi)^3\delta^{(3)}(\vec{p}-\vec{Q}_X)}{E_X+|\vec{p}\,|-m_\pi-M_N-i0}\\ &\times\frac{1}{2}\sum_{\sigma_p=\pm 1/2}\langle p(\vec{0},\sigma_p)|J^+_{5\mu}(0)|X\rangle\langle X|J^-_{5\nu}(0)|p(\vec{0},\sigma_p)\rangle\\ &+\text{crossed terms}\Big]. \qquad (10)\end{aligned}$$

Thus, the total contribution to the S–wave amplitude $\delta f^{(\gamma)}$ of π^-p scattering at threshold, caused by one photon in the intermediate state can be written as $\delta f^{(\gamma)}=\sum_X\delta f^{(X\gamma)}$, where the S–wave amplitude $\delta f^{(X\gamma)}$ is defined by the reaction

$\pi^- p \to X\gamma$. As we pointed out in the Introduction, there are strong reasons to believe that the physical contribution is nearly saturated by the one–baryon states $X = n$ and $X = \Delta(1232)$.

3. Contributions from the $\pi^- p \to n\gamma$ and $\pi^- p \to \Delta\gamma$ intermediate states

3.1. The $\pi^- p \to n\gamma$ contribution

This case corresponds to $X = n$. It has the physically open radiative decay channel, which gives rise to an imaginary part of the amplitude. This process is well understood. We will ignore it in the present discussion of contributions to the real part of the threshold amplitude. The contribution is given by the following sum and average over intermediate polarization states of the nucleon and photon.

$$\Big(1+\frac{m_\pi}{M_N}\Big)\delta f^{(n\gamma)} = \frac{1}{4}\frac{1}{F_\pi^2}\frac{\alpha}{M_N}\int\frac{d^3p}{(2\pi)^3 2|\vec{p}|}\sum_\lambda$$
$$\times\mathcal{P}\int\frac{d^3Q}{(2\pi)^3 2E_n(\vec{Q})}\,\frac{(2\pi)^3\,\delta^{(3)}(\vec{p}-\vec{Q})}{E_n(\vec{Q})+|\vec{p}\,|-m_\pi-M_p-i0}$$
$$\times\,\langle|\langle n(\vec{Q},\sigma_n)|e^*(\vec{p},\lambda)\cdot J_5^{(-)}(0)|p(\vec{0},\sigma_p)\rangle|^2\,\rangle_{av}\,. \quad (11)$$

The matrix element of the axial current between nucleon states is defined by [9,15]

$$\langle n(\vec{Q},\sigma_n)|J_{5\mu}^{(-)}(0)|p(\vec{0},\sigma_p)\rangle =$$
$$= g_A\,F_A(\vec{Q}^{\,2})\,\bar{u}_n(\vec{Q},\sigma_n)\gamma_\mu\gamma^5 u_p(\vec{0},\sigma_p), \quad (12)$$

where $g_A = 1.270(3)$. The axial form factor $F_A(\vec{Q}^{\,2})$ can be empirically approximated by

$$F_A(\vec{Q}^{\,2}) = (1+\vec{Q}^{\,2}/M_A^2)^{-2}, \quad (13)$$

where $M_A = (960\pm 30)\,\mathrm{MeV}$ (see e. g. [9,15]). The squared matrix element of the axial current, averaged over polarizations of the proton and summed over polarizations of the neutron is

$$\sum_\lambda\langle|\langle n(\vec{Q},\sigma_n)|e^*(\vec{p},\lambda)\cdot J_5^{(-)}(0)|p(\vec{0},\sigma_p)\rangle|^2\rangle_{av} =$$
$$4M_N^2 g_A^2(\vec{Q}^{\,2})\,\frac{1}{2}\sum_\lambda \mathrm{tr}\{(\vec{\sigma}\cdot\vec{e}^{\,*}(\vec{p},\lambda))(\vec{\sigma}\cdot\vec{e}(\vec{p},\lambda))\}$$
$$= 12M_N^2 g_A^2(\vec{Q}^{\,2}), \quad (14)$$

where the photon polarization is both transverse and longitudinal.

Since this is a perturbative contribution to the pion scattering length, we make for simplicity the heavy–baryon approximation in the following. The amplitude $\delta f^{(n\gamma)}$ becomes with $p = |\vec{p}|$

$$\Big(1+\frac{m_\pi}{M_N}\Big)\delta f^{(n\gamma)} =$$
$$\frac{3\alpha}{8\pi^2}\frac{g_A^2}{F_\pi^2}\,\mathcal{P}\int_0^\infty\frac{dp\,p\,F_A^2(p^2)}{p-m_\pi-i0}. \quad (15)$$

In the case of the form factor (13) the integral can be evaluated analytically. We define the function $I(x)$ with the even *(e)* and odd *(o)* parts $I^{(e,o)}(x) = (1/2)(I(x)\pm I(-x))$ as

$$I(x) = \mathcal{P}\int_0^\infty\frac{dt\,t}{t+x-i0}\frac{1}{(1+t^2)^4}. \quad (16)$$

Using the notation $d = (1+x^2)$ gives

$$I^{(e)}(x) = \frac{\pi}{2}\Big(\frac{1}{d^4}-\frac{1}{2d^3}-\frac{1}{8d^2}-\frac{1}{16d}\Big)\simeq$$
$$\simeq\frac{5\pi}{32}-\frac{35\pi}{32}x^2+\ldots \quad (17)$$

and

$$I^{(o)}(x) = x\Big(\frac{1}{2}\frac{\ln x^2}{d^4}+\frac{1}{2d^3}+\frac{1}{4}\frac{1}{d^2}+\frac{1}{6d}\Big)\simeq$$
$$\simeq x\Big(\frac{1}{2}\ln x^2+\frac{11}{12}+\ldots\Big). \quad (18)$$

Using these relations, the amplitude $\delta f^{(n\gamma)}$ in Eq. (15) can be expanded in the small parameter $x = x_\pi = m_\pi/M_A$:

$$\Big(1+\frac{m_\pi}{M_N}\Big)\delta f^{(n\gamma)} = \frac{3\alpha}{8\pi^2}\frac{g_A^2}{F_\pi^2}M_A I(-x_\pi) = \frac{3\alpha}{8\pi^2}\frac{g_A^2}{F_\pi^2}$$
$$\times\Big[\frac{5\pi}{32}M_A - m_\pi\Big(\ln\frac{m_\pi}{M_A}+\frac{11}{12}+\mathcal{O}(\frac{m_\pi}{M_A})\Big)\Big]. \quad (19)$$

This result applies identically to the correction of threshold amplitude for the charge symmetric reaction $\pi^+ n \to \pi^+ n$ with no change. The result for the scattering lengths for the crossed reactions $\pi^- n\to\pi^- n$ and $\pi^+ p\to\pi^+ p$ also follows immediately, since the only change is the replacement $m_\pi\to -m_\pi$ in the integral (15).

The contributions to the πN scattering amplitude at threshold from these radiative processes with a nucleon are thus

$$\Big(1+\frac{m_\pi}{M_N}\Big)\delta f^{(N\gamma)} =$$
$$= \frac{3\alpha}{8\pi^2}\frac{g_A^2}{F_\pi^2}M_A[I^{(e)}(x)t_3^2 + I^{(o)}(x)t_3\tau_3]\,. \quad (20)$$

Here t_3 and τ_3 are isospin matrices of pions and nucleons [5]. The amplitude (20) therefore does not contribute to reactions involving the neutral pion, including charge exchange.

An alternative derivation is to start from pseudovector πNN coupling with a Lagrangian term

$$\mathcal{L}_{\pi^- pn}(x) = \frac{g_A}{\sqrt{2}F_\pi}\,\bar{n}(x)\gamma^\mu\gamma^5 p(x)\partial_\mu\pi^-(x). \quad (21)$$

The result follows replacing $\partial_\mu \to \partial_\mu - ie\mathcal{A}_\mu$, where $\mathcal{A}_\mu$ is the e. m. vector potential. This treatment is analogous to that of the matrix element for the radiative capture process at threshold in the Panofsky ratio.

3.2. The $\pi^- p \to \Delta\gamma$ contribution

The S–wave amplitude of $\pi^- p$ scattering near threshold has contributions from the two intermediate states $\Delta^0\gamma$ and $\Delta^{++}\gamma$ which appear in the s– and u–channels of the reaction $\pi^- + p \to \Delta + \gamma \to \pi^- + p$, respectively. The corresponding two matrix elements are expressed as those of the axial currents $\langle\Delta^0|J^{(-)}_{5\mu}(0)|p\rangle$ and $\langle\Delta^{++}|J^{(+)}_{5\mu}(0)|p\rangle$. These are empirically known from the theoretical and experimental analysis of the neutrino production of the Δ^{++} resonance [16] by the reaction $\nu_\mu + p \to \mu^- + \Delta^{++}$ together with isospin invariance of the strong low–energy interaction. In the heavy–baryon limit the matrix elements $\langle\Delta^{++}|J^{(+)}_{5\mu}(0)|p\rangle$ and $\langle\Delta^0|J^{(-)}_{5\mu}(0)|p\rangle$ are defined as follows [16]:

$$\langle\Delta^{++}(\vec{Q},\sigma_\Delta)|J^{(+)}_{5\mu}(0)|p(\vec{0},\sigma_p)\rangle =$$
$$= +\sqrt{2}\,g_A\,F_A(\vec{Q}^{\,2})\,\bar{u}_{\Delta^{++}\mu}(\vec{Q},\sigma_\Delta)u_p(\vec{0},\sigma_p) \quad (22)$$

and

$$\langle\Delta^0(\vec{Q},\sigma_\Delta)|J^{(-)}_{5\mu}(0)|p(\vec{0},\sigma_p)\rangle =$$
$$= -\sqrt{\frac{2}{3}}\,g_A\,F_A(\vec{Q}^{\,2})\,\bar{u}_{\Delta^0\mu}(\vec{Q},\sigma_\Delta)u_p(\vec{0},\sigma_p). \quad (23)$$

Here $u_{\Delta^{++}\mu}(\vec{Q},\sigma_\Delta)$ and $u_{\Delta^0\mu}(\vec{Q},\sigma_\Delta)$ are wave functions of the Δ^{++} and Δ^0 resonances [9,15].

In the heavy–baryon limit the contributions to the S–wave amplitude of $\pi^- p$ scattering near threshold from the intermediate $\Delta^0\gamma$ and $\Delta^{++}\gamma$ states can be expressed using Eq. (16)

$$\Big(1+\frac{m_\pi}{M_N}\Big)\delta f^{(\Delta^{++}\gamma)} = \frac{\alpha}{2\pi^2}\frac{g_A^2}{F_\pi^2}M_A I(x_\Delta + x_\pi)$$
$$\Big(1+\frac{m_\pi}{M_N}\Big)\delta f^{(\Delta^{0}\gamma)} = \frac{\alpha}{6\pi^2}\frac{g_A^2}{F_\pi^2}M_A I(x_\Delta - x_\pi). \quad (24)$$

Here $x_\Delta = \omega_\Delta/M_A$ with $\omega_\Delta = M_\Delta - M_N$. The amplitudes $\delta f^{(\Delta^{++}\gamma)}$ and $\delta f^{(\Delta^0\gamma)}$ are calculated with the contributions of transverse and longitudinal photons.

The same result for the contribution to the S–wave amplitude of $\pi^- p$ scattering at threshold given by Eq.(24) follows from the effective Lagrangian of the $\pi\Delta N$–interactions [17]

$$\mathcal{L}_{\pi \mathrm{N}\Delta}(x) = \frac{g_A}{F_\pi}\Big[\bar{\Delta}^{++}_\omega(x)\Theta^{\omega\varphi}p(x)\partial_\varphi\pi^+(x)$$
$$-\frac{1}{\sqrt{3}}\,\bar{\Delta}^0_\omega(x)\Theta^{\omega\varphi}p(x)\partial_\varphi\pi^-(x) + \ldots + \mathrm{h.c.}\Big], \quad (25)$$

where $\Theta^{\omega\varphi} = g^{\omega\varphi} - (Z+1/2)\,\gamma^\omega\gamma^\varphi$ and Z is a parameter constrained by $|Z| \le 1/2$ [17], with the inclusion of the electromagnetic interaction by the minimal extension:

$$\mathcal{L}_{\pi \mathrm{N}\Delta\gamma}(x) = i\,\frac{eg_A}{F_\pi}\Big[\bar{\Delta}^{++}_\omega(x)\Theta^{\omega\varphi}p(x)\pi^+(x)$$
$$-\frac{1}{\sqrt{3}}\,\bar{\Delta}^0_\omega(x)\Theta^{\omega\varphi}p(x)\pi^-(x) + \ldots\Big]\mathcal{A}_\varphi(x). \quad (26)$$

In the heavy–baryon approximation the contribution of the Δ–resonance to $f^{\Delta\gamma}$ is independent of the parameter Z.

As previously for the case of $N\gamma$ intermediate states this result readily generalizes to any $\pi^\pm N \to \pi^\pm N$ elastic scattering amplitude. In the heavy Δ limit the contributions are charge symmetric. For example, the contribution to the $\pi^+ n$ scattering length is obtained from the $\pi^- p$ one of Eqs. (24) and (26) by the replacements $\Delta^0 \to \Delta^+$ and $\Delta^{++} \to \Delta^-$ with an unchanged result. In analogy to Eq.(20) we can write this result in terms of the even and odd functions in the variable x_π:

$$I^{(e,o)}_\Delta = [I(x_\Delta - x_\pi) \pm I(x_\Delta + x_\pi)]/2,$$

such that the total contribution from intermedi-

---

[5] t_3 is a diagonal 3×3 matrix with diagonal elements $(1,0,-1)$. In the non–diagonal representation it reads $(t_3)^{ab} = -i\epsilon^{ab3}$; $(t_3^2)^{ab} = \delta^{ab} - \delta^{a3}\delta^{b3}$ with $a,b = 1,2,3$

ate $\Delta\gamma$ states is

$$\Big(1+\frac{m_\pi}{M_N}\Big)\delta f^{(\Delta\gamma)} =$$
$$\frac{\alpha}{3\pi^2}\,\frac{g_A^2}{F_\pi^2}\,M_A\,[2I_\Delta^{(e)}t_3^2+I_\Delta^{(o)}t_3\tau_3]. \qquad (27)$$

We have based our evaluation in Table 1 directly on this expression. While it is possible to make a perturbative expansion in x_Δ and/or x_π as small parameters, such an expansion is poorly convergent and not very informative. This is in contrast to Eq. (20) for the nucleon case.

4. Discussion

In this section we investigate numerically the electromagnetic contributions to the scattering lengths of charged pions due to $N\gamma$ and $\Delta\gamma$ intermediate states. These intermediate states are associated with characteristic momenta of order $M_A/2 \simeq 500\ MeV/c$ or less. The results in terms of amplitudes are listed in Table 1, while we discuss the contributions in terms of % of the typical scale in the text [6].

The amplitudes can be expressed in terms of a general classification of isospin breaking amplitudes at threshold [10,18]

$$T_{\pi N}^{ba}(0) = 4\pi\left(1+\frac{m_\pi}{M_N}\right) f_{\pi N}^{ba}(0) =$$
$$= \delta^{ab}(g_{ba}^{+}+\tau^3 g_{ba}^{3+}) + i\epsilon^{bac}\tau^c(g_{ba}^{-}+\tau^3 g_{ba}^{3-}). \qquad (28)$$

Inside of this classification the terms generated by the present mechanism by Eqs. (19,20) and (27) correspond predominantly to a large isospin violating contribution proportional to t_3^2, which contributes to the term g_{ab}^{+}. To leading order in the isospin breaking amplitude such a term appears also in the heavy baryon ChPT expansion and has the form [10]

$$\delta g_{ba}^{+} = -8\pi\,\alpha\, f_1\,(\delta^{ab}-\delta^{a3}\delta^{b3}), \qquad (29)$$

where f_1 is a ChPT constant. The sign of this term is not known, but a dimensional estimate gives $F_\pi^2|f_1| \simeq M_N/16\pi^2 = 6$ MeV [10]; another estimate doubles this value to 12 MeV [7]. The amplitudes defined by Eqs. (19) and (27) give a corresponding term in the limit $m_\pi = 0$:

$$F_\pi^2 f_1 = -\frac{15 g_A^2}{512\pi}\,M_A\Big(1+\frac{16}{9}\,\frac{I^{(e)}(\omega_\Delta/M_A)}{I^{(e)}(0)}\Big). \qquad (30)$$

This result is not an evaluation of the constant f_1 within the framework of ChPT and EFT, but rather means that we have identified the main physical mechanism which leads to such a constant. The dimensional magnitude estimates quoted above with a large value are roughly consistent with our contribution of -9.3 MeV from the $N\gamma$ channel alone without the contribution from the Δ isobar, which gives a substantial increase. This isospin-breaking term contributes equally to each of the 4 elastic πN amplitudes. The effect is quite large. We now discuss the contributions in more detail. It is convenient to discuss them in % of the experimental $\pi^- p$ scattering length $a_{\pi^- p} = 0.0883 m_\pi^{-1}$ [2], which sets the scale for the elastic threshold amplitudes of charged pions. In particular, while the dimensional EFT estimate gives a $\pm1.4\%$ to $\pm2.8\%$ contribution from f_1 to the level shift in the π^-p atom [7], our result of Table 1 gives a $+6.1(3)\%$ contribution in the limit $m_\pi = 0$.

For physical pions, the intermediate $N\gamma$ state alone contributes 3.4% to the scattering amplitude, which increases to 9.4% when the Δ resonance is included on an equal footing and degenerate with the nucleon (see Table 1). The reason for these surprisingly large numbers is that the present mass scale is larger by $M_A/m_\pi \simeq 7$ than the chiral one. The e. m. contribution based on this term is therefore increased substantially. The attractive sign and magnitude is general, while the detailed prediction depends on physical assumptions. In particular, the physical $N\Delta$ mass difference quenches the Δ contribution from 5.6% to 2.7%. Any realistic evaluation of the constant f_1 must include the Δ isobar as well as the $N\Delta$ mass splitting.

In the EFT treatment the constant f_1 contributes also to the nucleon e. m. mass, which is outside the present approach. Even so this raises an interesting problem. In our case the corre-

[6] For numerical analysis we set $\omega_\Delta = 290\,\mathrm{MeV}$; $\omega_\Delta + m_\pi = 430\,\mathrm{MeV}$ and $\omega_\Delta - m_\pi = 150\,\mathrm{MeV}$. Note that in the soft–pion limit the mass difference $\omega_\Delta \neq 0$.

sponding f_1 in the πN sector is heavily dominated by the axial coupling such that it is a magnitude larger than the value expected on the basis of the residual nucleon e. m. mass term beyond the part due to the np mass difference. However, the average nucleon e. m. mass in the EFT approach does not depend on f_1 but on the sum of *two* ChPT constants $(f_1 + f_3)$ These are not separate observables in the nucleon sector [7]. This suggests that axial contributions in the two terms may cancel in the sum, at least to leading order. A recent study of the e. m. corrections in a heavy–quark model [11] indicates that this may indeed be the case. The three ChPT constants (f_1, f_2, f_3) linked to e. m. effects are explicitly evaluated in this model. Although not stated by the authors, the total e. m. nucleon mass proportional to $(f_1 + f_3)$ depends only on the nucleon charge and magnetic form factors in their model. Their f_1 constant, on the other hand, is dominated by the axial form factor, as we also find. A massive cancellation in the sum $f_1 + f_3$ eliminates the axial form factor from the nucleon e. m. mass term. There is then no contradiction between the chiral result with a contribution to the e. m. nucleon mass, and ours with no contribution to that sector. While the physical ingredients differ substantially, our heavy baryon result is $F_\pi^2 f_1 = -25.8(8)$ MeV as compared to the heavy quark model value $-19.5(1.6)$ MeV.

In addition to this leading contribution our mechanism generates characteristic terms depending on the non–vanishing pion mass, which are counterparts to f_1. Once more, such terms give contribution neither to the charge exchange amplitude nor to the neutral pion scattering one at threshold. They introduce in particular an isospin breaking term proportional to $t_3\tau_3$. In an EFT expansion such terms have to our knowledge only been considered in Ref. [7], where they occur as a 3rd order term. They are outside the leading order isospin breaking considered in Refs. [10,18]. The reason for these terms is that the pion mass contributes with opposite sign in the denominators for a direct as compared to a crossed process.

Consider the case of the $\pi^- p$ scattering length. In the case of the $n\gamma$ intermediate state, the following terms of order m_π and $m_\pi \ln m_\pi$ from Eq. (19) contribute:

$$\mathcal{T}^{(n\gamma)} \equiv 4\pi \left(1 + \frac{m_\pi}{M_N}\right)\delta f^{(n\gamma)} =$$
$$-\frac{3\alpha}{2\pi}\frac{g_A^2}{F_\pi^2}\, m_\pi \Big(\ln \frac{m_\pi}{M_A} + 0.917 + \mathcal{O}(\frac{m_\pi}{M_A})\Big). \tag{31}$$

(for numerics see Table 1, where also the next order terms are included). This term has an exact counterpart to the same order in Ref. [7] where it is denoted by $\mathcal{T}_3^{em}(0)$ in the e. m. chiral power expansion of the $\pi^- p$ scattering length (their Eq. (9.5):

$$\mathcal{T}_3^{em}(0) \equiv 4\pi\Big(1 + \frac{m_\pi}{M_N}\Big)\delta f_3^{(em)} =$$
$$-\frac{3\alpha}{2\pi}\frac{g_A^2}{F_\pi^2}\, m_\pi \Big(\ln \frac{m_\pi}{\mu} + 0.891 + \mathcal{O}(m_\pi)\Big), \tag{32}$$

where $\delta f_3^{(em)}$ is the corresponding contribution to the scattering length. The leading $m_\pi \ln m_\pi$ is identical in the 2 cases as expected and independent of the axial mass M_A and the dimensional regularization mass μ. The term proportional to m_π cannot readily be compared although the dependence on M_A and μ is weak, since the relation of these scales to each other is not specified. Even so it is clear that the results are reminiscent as to sign, structure and magnitude.

The Δ isobar changes the character of the terms dependent on the pion mass. If we attempt to neglect the $N\Delta$ mass difference, the value of the isospin breaking isoscalar amplitude is increased dramatically by a factor 25/9 with respect to the value for the nucleon only. The isovector contribution on the other hand becomes negligible as a consequence of a nearly complete cancellation between the nucleon and Δ terms. This is a mathematical artifact, however, which does not correspond to the actual physics of the problem. Once the $N\Delta$ mass difference is introduced, the Δ term is substantially quenched in both cases. As a consequence the total isovector contribution from the joint $N\gamma$ and $\Delta\gamma$ states is now a modest -0.57% net contribution. This is nearly half of the result for the nucleon only in the absence of the mass splitting. The Δ degree of freedom including the $N\Delta$ mass difference are essential to the understanding of these terms.

Table 1
Contributions $10^3\, m_\pi(\delta f^{(N\gamma}+\delta f^{(\Delta\gamma)})$ in the heavy baryon limit to the πN scattering lengths corresponding to Eqs. (20) and (27). The quoted uncertainty reflects the one of M_A.

$m_\pi = 0, \omega_\Delta = 0$	$(3.0(1)_{N\gamma} + 5.3(2)_{\Delta\gamma})\, t_3^2$	$= 8.3(3)\, t_3^2$
$m_\pi = 0, \omega_\Delta \neq 0$	$(3.0(1)_{N\gamma} + 2.4(1)_{\Delta\gamma})\, t_3^2$	$= 5.4(2)\, t_3^2$
$m_\pi \neq 0, \omega_\Delta = 0$	$(2.6(1)_{N\gamma} + 4.6(1)_{\Delta\gamma})\, t_3^2 + (-0.8_{N\gamma} + 0.7_{\Delta\gamma})\, t_3\tau_3$	$= 7.2(2)\, t_3^2 + (-0.1)\, t_3\tau_3$
$m_\pi \neq 0, \omega_\Delta \neq 0$	$(2.6(1)_{N\gamma} + 2.5(1)_{\Delta\gamma})\, t_3^2 + (-0.8_{N\gamma} + 0.3_{\Delta\gamma})\, t_3\tau_3$	$= 5.1(2)\, t_3^2 + (-0.5)\, t_3\tau_3$

Our results are of direct relevance both to the determination of the πNN coupling constant as well as to that of the $\sigma_{\pi N}$–term. In the first case our results indicate that the e. m. contribution to the hadronic scattering length combination $(a_{\pi^- n} - a_{\pi^- p})$, which dominates the GMO dispersion relation for $g^2_{\pi NN}$ is only -0.3%, such that the evaluation in Refs. [6,19] remains nearly unchanged. Concerning the $\sigma_{\pi N}$–term the low–energy isospin symmetric $\pi^c N$ amplitude for charged pions extrapolated to the Cheng–Dashen point depends linearly on the corresponding isoscalar scattering length. The correction for the present e. m. contribution diminishes the value for $\sigma_{\pi N}(2m_\pi^2)$ by about 5 MeV, which is a sizable fraction of the major correction of 15 MeV associated with the further extrapolation to $t = 0$ so as to obtain $\sigma_{\pi N}(0)$ [20,21].

It may soon be possible to demonstrate the isospin symmetry breaking directly for the isospin odd amplitude in spite of its small value. This is of particular interest, since it depends both on the chiral isospin breaking in the strong sector [10,18] as well as on the present breaking for which the dependence on the form factor is expected to be weak. It requires a combination of precision measurements of the 1s level shifts in pionic hydrogen $(a_{\pi^- p})$ and in pionic deuterium $(a_{\pi^- p} + a_{\pi^- n})$ as well as the charge exchange 1s width in pionic hydrogen $(a_{\pi^- p\to\pi^0 n})$ [1,19]. The first two serve to eliminate the large isospin breaking in the isoscalar amplitude due to f_1, while the width gives the corresponding charge exchange amplitude.

5. Conclusion

We have previously investigated the isospin violating corrections to the πN scattering lengths induced by the external Coulomb field of the extended charge [6]. These can be well understood in physical terms to the present level of experimental precision. Here we have investigated the intrinsic isospin breaking in the πN scattering lengths induced by radiative capture processes with nucleon and Δ isobar intermediate states. The effect is large and intimately related to p-wave πN physics. In view of the obvious physical origin, there is no reason to believe that it will be suppressed by systematic cancellations in a more detailed treatment. The result has no free parameters. The most important violation occurs in the isoscalar scattering length for charged pions. This violating term has the same symmetry property as the one associated with the ChPT constant f_1. It follows from our result that the evaluation of this parameter requires that both the nucleon and the Δ isobar are taken into account as well as their mass difference. In addition, their axial form factor must be included in accordance with observations. The finite pion mass generates small isospin breaking terms, mainly in the isovector amplitude. The nucleon gives to leading order in the pion mass a term proportional to $m_\pi \ln m_\pi$ with a coefficient identical to the one obtained previously for the $\pi^- p$ system using EFT [7]. However, the physics of this isospin breaking is governed by the interplay of the p-wave πN contributions from both nucleons and Δ isobars and has not been previously investigated. The $\Delta\gamma$ intermediate state gives a contribution from the physical pion mass of opposite

sign, which largely cancels the corresponding nucleon term. Here the NΔ mass difference and the pion mass enter in a non-linear combination. The result is a modest net isospin violation in the isovector amplitude.

The importance of our +6.3 % contribution to the energy shift in the 1s state of pionic hydrogen is evident when compared to previous results. Sigg et al. [24] find a $-2.1 \pm 0.5\%$ overall correction with an *ad hoc* estimate of only -0.7% for the $N\gamma$ contribution and the sign is opposite to ours. Gasser et al. [7] find an overall correction of $-7.2 \pm 2.9\%$ in which the error comes nearly entirely from the uncertainty in f_1. This error is estimated dimensionally as a one photon loop term, which gives a magnitude of about half of our f_1 and of unknown sign.

Our description can be improved beyond the heavy–baryon approximation along similar lines as here. We expect a kinematic decrease of the leading isospin violating term, but no qualitative change. However, such improvements will give small charge symmetry violating terms.

Our mechanism for isospin breaking in elastic πN scattering gives contributions to the 1s level shift in pionic hydrogen 30 times larger than present experimental precision and it is a central feature in the breaking of isospin symmetry for πN scattering at threshold. So as to optimally exploit the high precision of ongoing experiments on pionic hydrogen and deuterium [1,2], it is therefore desirable to refine the present results.

Acknowledgments

Torleif Ericson is grateful to Profs. P. Kienle, J. Marton and M. Faber for their hospitality and Prof. B. Loiseau for useful discussions. Andrei Ivanov thanks the TH–division at CERN for hospitality during part of this work. The present collaboration originated from a discussion at a CPT* Workshop at Trento in October of 2003.

REFERENCES

1. D. Gotta *et al.*, Physica Scripta, T **104**(2003) 94; hep-ex/0305012.
2. H.–Ch. Schröder *et al.*, Eur. Phys. J. C **21** (2001) 473.
3. S. Deser, *et al.*, Phys. Rev. **96** (1954) 774.
4. T. L. Trueman, Nucl. Phys. **26** (1961) 57.
5. J. Spuller *et al.*, Phys. Lett. B **67** (1977) 497.
6. T. E. O. Ericson, B. Loiseau, and S. Wycech, Phys. Lett. B **594** (2004) 76.
7. J. Gasser, *et al.*, Eur. Phys. J. C**26** (2002) 13.
8. N. M. Kroll and M. A. Ruderman, Phys. Rev. **93** (1954) 233.
9. T. E. O. Ericson and W. Weise, in *Pions and Nuclei*, Clarendon Press, Oxford, 1988.
10. N. Fettes and U.–G. Meißner, Phys. Rev. **C 63** (2001) 045201; (hep-ph/0008181).
11. V. E. Lyubovitskij, *et al.*, Phys. Rev. **C 65** (2002) 025202.
12. V. E. Lyubovitskij and A. Rusetsky, Phys. Lett. B **494** (2000) 9.
13. S. L. Adler and R. Dashen, in *Current Algebras* , Benjamin, New York 1968.
14. M. Ericson and M. Rho, Phys. Rep. **5** (1972) 57.
15. M. M. Nagels *et al.*, Nucl. Phys. **B147** (1979) 189.
16. S. Berman and M. Veltman, Nuo. Cim. **38** (1965) 993; P. A. Schreiner and F. von Hippel, Phys. Rev. Lett. **30** (1973) 339; G. F. Brown and W. Weise, Phys. Rep. **22** (1975) 279.
17. L. M. Nath, *et al.*, Phys. Rev.**D3** (1971) 2153; M. G. Olsson and B. T. Osypowski, Nucl. Phys. **B87** (1975) 399; V. Bernard, et al., Int. J. Mod. Phys. E **4** (1995) 193. A. N. Ivanov, M. Nagy, and N. I. Troitskaya, Phys. Rev. **C59** (1999) 451.
18. N. Fettes, *et al.*, Phys. Lett. B **451** (1999) 233.
19. T. E. O. Ericson, B. Loiseau, and A. W. Thomas, Phys. Rev. **C66** (2002) 014005.
20. P. Schweizer, Eur.Phys.J. **A22** (2004) 89.
21. J. Gasser, H. Leutwyler and M. E. Sainio, Phys. Lett. B **253** (1991) 252 qnd 260.
22. T. R. Hemmert, et *et al.*, J. Phys. G:Nucl. Part. Phys. **24** (1998) 1831.
23. N. Fettes and Ulf–G. Meißner, Nucl.Phys. A**A679** (2001) 629.
24. D. Sigg, et al., Nucl. Phys. **A 609** (1996) 310.